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A 3-D Fokker-Planck Code for Studying Parallel Transport in Tokamak Geometry with Arbitrary Collisionalities and Application to Neoclassical Resistivity

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Abstract

A new 3-D Fokker-Planck code, $CQL_{||}$, which solves the Fokker-Planck equations with two velocity coordinates and one spatial coordinate parallel to the magnetic field lines **B**/B, has been developed. This code enables us to model the parallel transport for low, intermediate and high collisional regime. The physical model, the possible relevant applications of the code as well as a first application, the computation of the neoclassical resistivity for various collisionalities and aspect ratios in tokamak geometry are presented.

1. Introduction

There are numerous situations in tokamak plasma transport and divertor physics which are in an intermediate collisional regime, where the mean-free path λ_{mfp} is comparable to plasma parameter scale-lengths $L_{||}$ along the magnetic field lines. Neither fluid nor low collisionality theory is fully applicable. To study this class of problems we are developing a new Fokker-Planck (FP) code $CQL_{||}$ which solves the FP equations with two velocity coordinates (u, momentum per rest mass, and θ, pitch-angle), and one spatial coordinate, s, along the magnetic field lines **B**/B. $CQL_{||}$ is constructed building on the CQL3D code [1] which solves the FP equations with the same velocity coordinates, but the spatial coordinate is the plasma radius r rather than s. The diverse features of CQL3D are kept; multi-species, nonlinear, relativistic, toroidal geometry, non-circular flux surfaces. The bounce-averaging along **B** performed in CQL3D has been removed in $CQL_{||}$, allowing us to model arbitrary collisional regime. The equation is coupled either to quasi-neutrality condition or to Poisson equation, in order to calculate iteratively the electric field component along **B**, $E_{||}$, in a consistent way. Finite differences and an alternating direction implicit method (ADI) are used to solve the equation. The main application of this new code is in edge physics, where λ_{mfp} is often comparable to the scale-length of the temperature gradient, for example, and also the sheath and pre-sheath problems near the divertor plate.

In the following we shall describe the physics problem, the numerical method used and list possible applications of $CQL_{||}$. We shall also present a few benchmarks and a comparison of the neoclassical resistivity for various finite collisionalities ν_e^* and inverse aspect ratio ε obtained with $CQL_{||}$ with the formula given in Ref. 2 by Hinton and Hazeltine (HH formula).

2. Physics Model and possible applications

We use the Fokker-Planck equation, as described in Ref. 1, to model the evolution of the distribution function when collision and different forces, sources and sinks are present. The geometry of the plasma can range from a uniform magnetic field 1-D (one spatial dimension) plasma to non-up/down symmetric noncircular cross-section, axisymmetric toroidal plasma on a single flux surface.

The fully nonlinear effects of collisions are modeled using the Rosenbluth potentials. Alternatively, we assume a partially non-linear collision operator in which the evolved species (called "general" species hereafter) collide with a fixed background plasma made of species having shifted Maxwellian distribution conserving momentum, and with a consistent density variation along $\mathbf{B}$. We can study the evolution of one or several "general" species, electrons and/or ions, having arbitrary distribution function $f_a(u, \theta, s; t)$, interacting with the background plasma and in between themselves. All species may be "general" which corresponds to the fully nonlinear problem.

The poloidal electric field and its component along $\mathbf{B}$, which is created by a non-zero charge density due to the loss of the charges along the magnetic field lines, is calculated from the Poisson equation or alternatively according to charge neutrality. In addition, we can include an external toroidal or radial electric field ($\mathbf{E}_{\phi,\mathrm{ohmic}}$; $\mathbf{E}_r$).

We consider the gyro-averaged distribution function, assume zero Larmor radii, zero banana width and axisymmetric geometry. The equation is of the form:

$$\frac{\partial f}{\partial f} = C(f) + E(f) + Q(f) + S(f) + A(f) \qquad (1)$$

where $f=f(u,\theta,s;t)$, $u=\gamma v$, v is speed, and $\gamma=\sqrt{1+u^2/c^2}$. C, E, Q, and S are operators in the momentum space giving the diffusion and convection due to collisions, the effect of the electric field, the rf wave-fields, and additional sources/sinks, respectively [1]. The term A(f) given by:

$$A(f) = -\frac{u_\perp}{2}\frac{\psi'}{\psi}\frac{\partial f}{\partial \theta} - u_{||}\frac{\partial f}{\partial s},$$

with $\psi(s)=B(s)/B(s=0)\equiv B(s)/B_0$, reflects the convection along the magnetic field line. Integrating Eq. (1) over the velocity space we obtain:

$$\partial n/\partial t + \nabla \cdot \int d^3\mathbf{u}'\, \mathbf{u}_{\|}\, f = \int d^3\mathbf{u}'\, S(f)$$

That is, if there are no sources or sinks we recover the continuity equation as $\nabla \cdot \int f\mathbf{u}_{\perp} d^3\mathbf{u}$ is zero in our model. In the code we introduce θ_0=arcsin $(\sin\theta/\sqrt{\psi})$, such as to reduce A(f) to:

$$A(f) = -\, u \cos\theta(\theta_o,s) \left.\frac{\partial f(u,\theta(\theta_0,s),s,t)}{\partial s}\right|_{\theta_0} . \qquad (2)$$

Note also that integrating Eq. (1) over q $u_{\|}$ $d^3\mathbf{u}$ (first moment) and using the charge neutrality condition, we can obtain an equation for $E_{\|}$.

The most relevant applications of such a code are in the plasma edge and divertor physics. A first application will be to assess the problem of the parallel heat flux in a collisionless or intermediate regime, which would quantify better the results given by Khan and Rognlien [3]. This code will also enable us to better understand the physics of parallel heat and particle transport near the divertor plate, the sheath and pre-sheath problems and numerous other problems directly relevant for the design and understanding of divertors as they are often in an intermediate collisional regime.

Another set of applications will be the comparison with standard neoclassical transport and extension of the existing formulae, for example for resistivity or bootstrap current, to finite inverse aspect ratio ε and collisionality ν_e^* and to real tokamak geometry.

3. Numerical method

We use the same finite difference method as in CQL3D [1] to approximate the diffusion operator in velocity space. The Rosenbluth potentials are also computed using a Legendre decomposition. To approximate A(f) in the form of Eq. (2) we use a simple forward/backward implicit method depending on the sign of cos θ ($u_{\|}<0$ or $u_{\|}>0$, respectively), or a centered implicit scheme. In order to solve Eq. (1) while keeping a maxmimum 2-D dimension for the solvers, we use an alternating direction implicit method (ADI) as follows:

$$\frac{f^{n+1/2}-f^{\,n}}{\Delta t/2} = [C + E + Q + S]\,(f^{\,n+1/2}) + A(f^n) \qquad (3a)$$

$$\frac{f^{n+1}-f^{\,n+1/2}}{\Delta t/2} = [C + E + Q + S]\,(f^{n+1/2}) + A(f^{n+1}) \qquad (3b)$$

If the electric field is computed using the Poisson equation or the charge neutrality condition, then it can be determined in between time-steps or half time-steps. In the following we use a circular plasma with zero Shafranov shift and a regular mesh along s with s=0 at the outer midplane. Thus the point l=ls/2+1 is at the inner midplane and the point l=ls+1 coincides with l=1 (periodicity). The theta mesh is constructed such that the points (θ,l) lie on μ=cst lines which enables us to easily calculate the term A(f) in Eq. (2). However, it means that there are less θ points at l=ls/2+1 than at l=1.

4. Results

In this paper we present only results obtained with a fixed ohmic electric field ($E\phi \sim 1/R$) and we assume Q=S=0 and no electrostatic electric field. As a first benchmark we have calculated the value of the parallel resistivity with respect to the Spitzer resistivity η_{sptz} at various values of s when the transport is turned off. That is, the different s positions are independent and they should all have $\eta/\eta_{sptz}=1$ as there are no trapped particles. We obtain 0.99 for each s with some minor differences near $l \approx ls/2$, at the inner midplane, where there are less θ–mesh points.

As a second step we turn the parallel transport on and compute the neoclassical resistivity, $\eta_{CQL||} = \langle E_{||}B\rangle/\langle j_{||}B\rangle$, which we compare for different ε and ν_e^* with the resistivity obtained with CQL3D [1], η_{CQL3D} , and with the Hinton-Hazeltine formula, η_{HH} (Ref 2, eqs.6.122 and 6.126 with $Z_{eff}=1$), which we modify, in order to have the correct limit $\varepsilon \to 1$, as follows:

$$\eta_{HH}/\eta_{Sptz} = \frac{1}{1 - K_{33}\sqrt{\varepsilon} + (K_{33} - 1)\,\varepsilon} \tag{4}$$

$$K_{33} = 1.83\,(1 + 0.68\sqrt{\nu_e^*} + 0.32\,\nu_e^*)^{-1}\,(1 + 0.66\,\nu_e^*\,\varepsilon^{3/2})^{-1}$$

We have added the term proportional to ε in Eq.(4). We use the following plasma parameters: T_e = 10 keV and $n_e = 10^{11}$ cm^{-3}, such that we are in the zero collisionality limit as $\nu_e^* \le 10^{-4}$, and we vary the inverse aspect ratio ε. In this case, $CQL_{||}$ should recover the results of CQL3D, for all ε and those of Hinton-Hazeltine for small ε. This is clearly seen on Fig. 1 where the maximum relative error with CQL3D is 7%. Note that the HH formula is very good for large ε as well. This is due to the modification of the original formula, otherwise Eq.(6.122)[2] gives $\eta_{HH}<0$ for $\varepsilon>0.3$, and to the fact that we have a zero-shifted equilibrium and that the coefficient of $\sqrt{\varepsilon}$ in η_{HH} is 1.83 instead of 1.95 as in the analytic formula.

As a third benchmark, we keep $\varepsilon = 0.06$ fixed and vary ν_e^*, changing T_e from 10 to 0.1 keV and n_e from 10^{11} to 10^{14} cm^{-3}. The results are shown on Fig. 2 and we see that $\eta_{CQL||}$ follows well, within 8%, η_{HH}. We plot η_{CQL3D} to point out that, as the equations are bounce-averaged, it can only model the $\nu_e^* \cong 0$ limit. We have also plotted the original η_{HH} formula to show that even for ε=0.06, it makes a noticeable 10% difference.

Fig. 3 shows $\eta/\eta_{Sptz}\,(\nu_e^*)$ for two different ε. Note that we have changed the x-axis compare with Fig.2, in order to remove the direct dependence of ν_e^* on ε. As on Fig.2, we clearly see the three different regimes: banana, $\nu_e^*\varepsilon^{3/2} \le 10^{-2}$, plateau, $10^{-2} \le \nu_e^*\varepsilon^{3/2} \le 1\text{-}10$, and Pfirsch-Schlüter, $10 \le \nu_e^*\varepsilon^{3/2}$. We have seen on Fig.1, that the η_{HH} formula reproduces correctly the effect of large ε on the neoclassical resistivity. Fig.3 confirms this results for finite ε and arbitrary ν_e^*, as the two curves obtained from $CQL_{||}$ and HH are very similar for both ε. However we have found in a seperate study, assuming $\nu_e^*=0$, that the HH formula should be slightly modified for $\varepsilon \geq 0.2$ when equilibria with non-negligible Shafranov shifts are considered. Thus, we shall consider these equilibria with finite ν_e^* as well in a different study.

5. Conclusion

We have developed a 3-D Fokker-Planck code $CQL_{||}$ which can model the transport along the magnetic field line for arbitrary collisional regime and axysimmetric toroidal geometry. We have shown that it gives the correct results for the neoclassical parallel resistivity in the two known limits, that is for $\nu_e^*=0$ and arbitrary ε, comparing with CQL3D, and for $\varepsilon \approx 0$ and arbitrary ν_e^*, comparing with the Hinton-Hazeltine formula. Moreover, we have shown that for finite ε, up to 0.4, the Hinton-Hazeltine formula is still valid for arbitrary ν_e^*, at least with a circular non-shifted equilibrium. Further studies, using different equilibria, are underway to better characterize the domain of validity of this formula.

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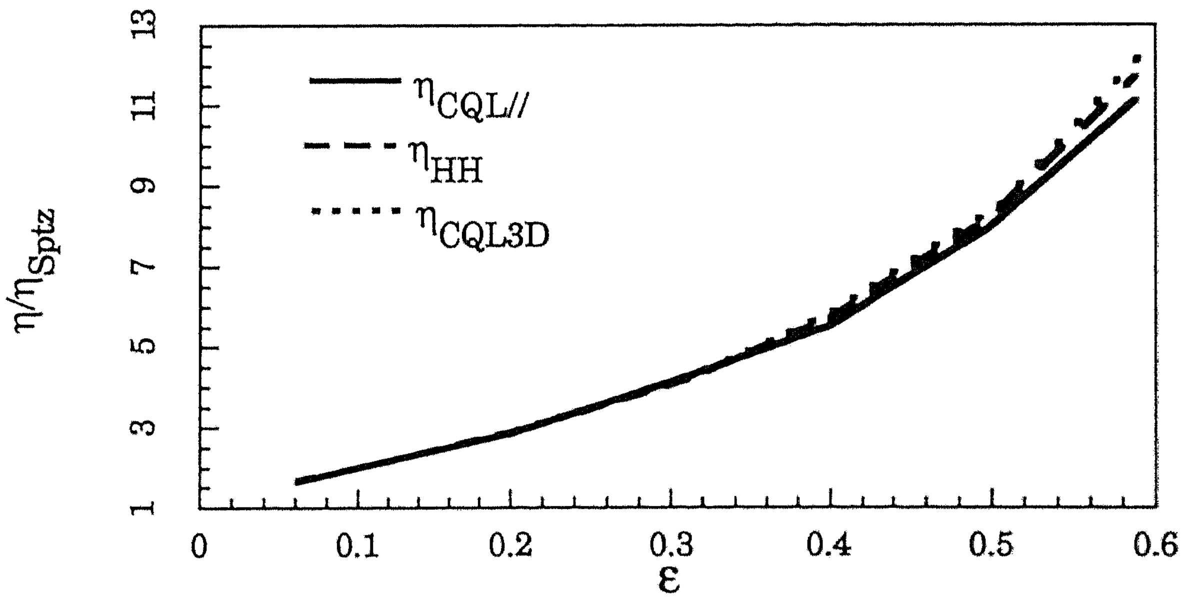

Fig.1: Resistivity versus ε for $n=10^{11}cm^{-3}$ and T=10keV, i.e. $\nu_e^* \leq 10^{-4}$.

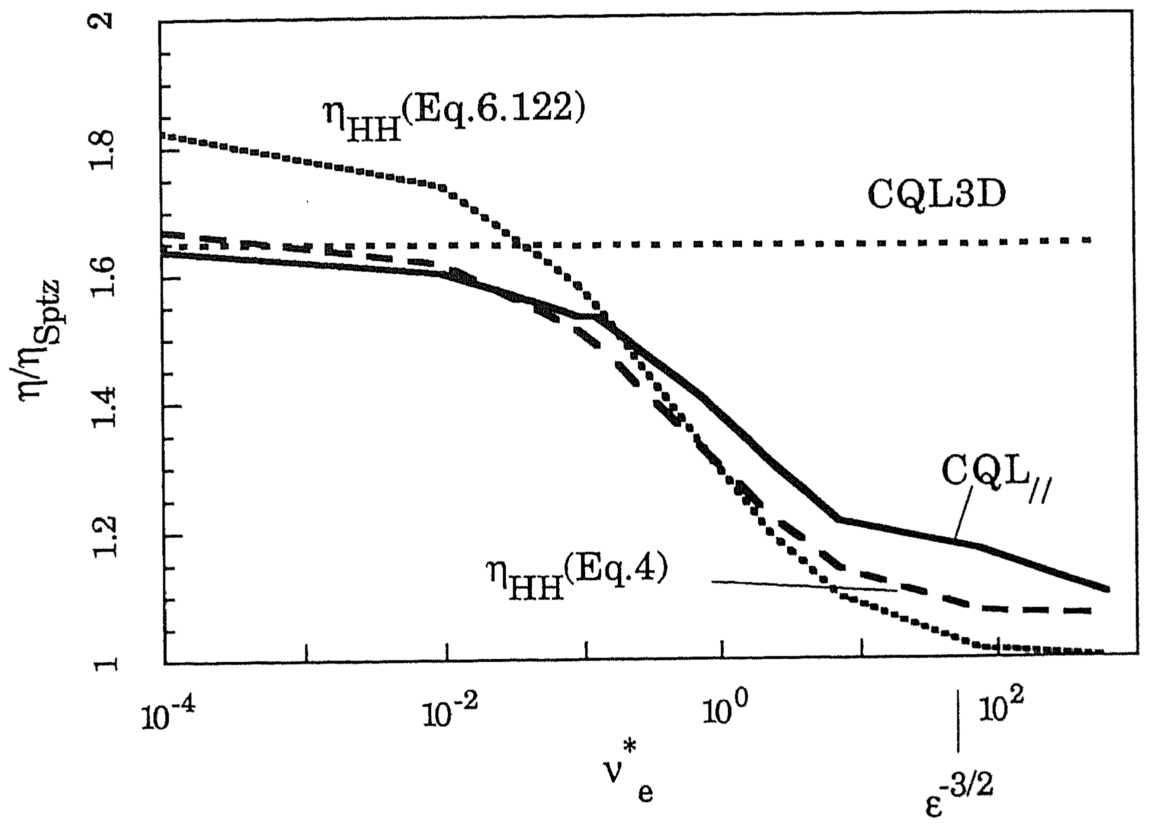

Fig. 2: Resistivity vs. ν_e^* for ε=0.06, with CQL//, CQL3D, Eqs.(4) and (6.122)[2].

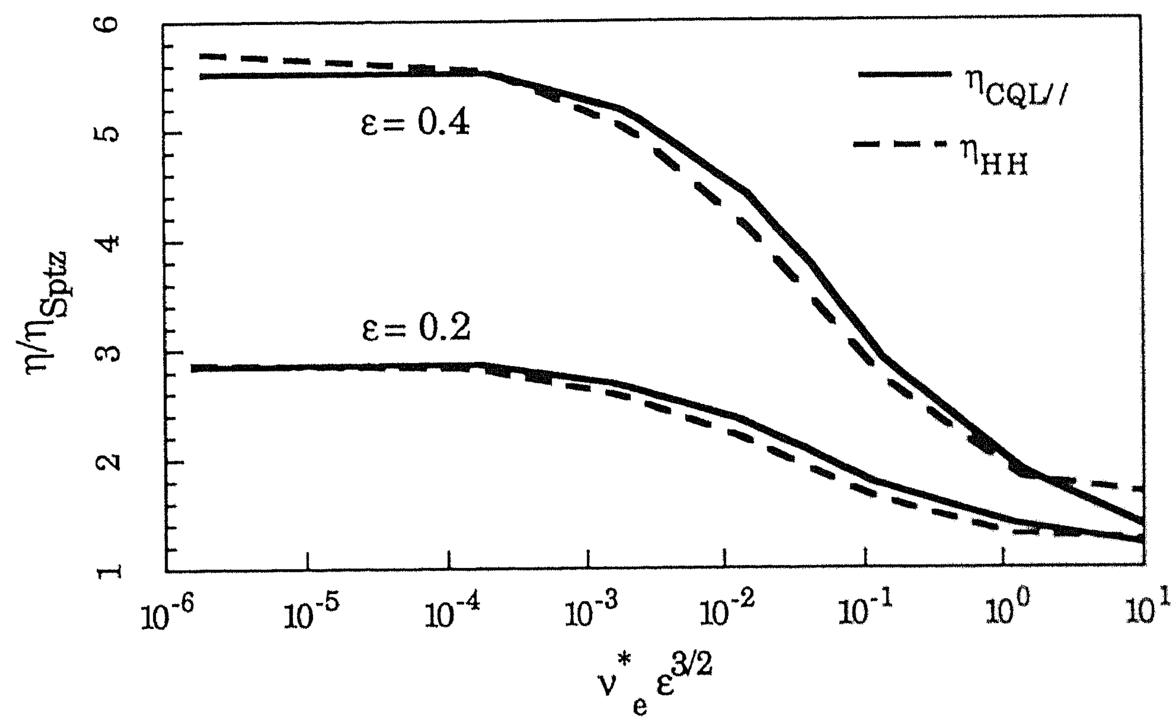

Fig. 3: Resistivity vs. ν_e^* $\varepsilon^{-3/2}$ for ε=0.2 and ε=0.4, using CQL// and Eq.(4).