VI) Electron Cyclotron Emission

Major diagnostic for Te profiles on most fusion devices. Provides info on suprathermal (HFS ECE on TCV).

Electron cyclotron frequency;

\[ \Omega_{ce} = \frac{eB}{\gamma m_e} \]

\( \gamma m_e \) is the relativistic mass

\[ \gamma = \frac{1}{\sqrt{1-v_e^2/c^2}} \]

Larmor radius \( \rho_e = \left| \frac{v_e}{\Omega_{ce}} \right| \)

Power emitted by an electron accelerated by Lorentz force is given by Schott-Trubnikov formula (see Hutchinson, Principles of Plasma Diagnostics):

\[ \frac{d^2P}{d\omega d\Omega} = \frac{e^2c^2}{8\pi^2e_0^2e^2} \sum_{m=1} \left\{ \left( \frac{\cos \theta - \beta_\parallel }{\sin \theta} \right)^2 J^2_m(\beta) + \beta_\parallel^2 J^2_0(\theta) \right\} \times \frac{\delta\left(\omega - m\Omega_{ce}\right)}{1 - \beta_\parallel^2 \cos \theta} \]

\[ \beta_\parallel = \frac{V_\parallel}{c}, \quad \beta_\perp = \frac{V_\perp}{c}, \quad \beta = \frac{V}{c}, \quad J_m = \frac{m\beta_\parallel \sin \theta}{1 - \beta_\parallel^2 \cos \theta} \]

This is a line spectrum with lines at the cyclotron harmonics:

\[ \omega_m = \frac{m\Omega_{ce}}{1 - \beta_\parallel \cos \theta} \]
Plasma emissivity is obtained by integrating over distribution function:

\[ j(\omega, \theta) = e^3 \int \frac{d^4p}{d\omega d\Omega} \left( 1 - \beta_x \cos \theta \right) \tilde{f}(\beta_x, \beta_y) \frac{2\pi \beta_x \beta_y}{2\pi \beta_x \beta_y} d\beta_x d\beta_y \]

where \( \tilde{f}(\beta_x, \beta_y) = \tilde{f}(x/c) \)

For thermal electrons at harmonic m:

\[
\begin{align*}
\dot{j}_m &= \frac{e^2 \omega_m^3 n_e}{8\pi^2 \varepsilon_0 c} \frac{m^{m-1}}{(m-1)!} \sin \theta \left( \frac{2 T_e}{m^2 e c^2} \right)^m \frac{\omega}{m^2 \Delta \omega} \\
\end{align*}
\]

These lines are broadened by two main mechanisms: relativistic broadening and Doppler broadening (collisional and radiative broadening are negligible)

\[
\frac{\Delta \omega}{\omega_m} \text{relat} \approx \frac{T_e}{m_e c^2} \quad \text{non-Gaussian shape}
\]

\[
\frac{\Delta \omega}{\omega_m} \text{Doppler} \approx \sqrt{\frac{2 T_e}{m_e c^2}} \cos \theta \quad \text{Gaussian}
\]
Relation between emission and absorption

Transport of electron cyclotron emission is dominated by a balance of emission and reabsorption. Consider an e-m wave (or ray) propagating from a point $S_0$ to $S$ in a homogeneous medium of temperature $T$ ($T_e$ in a plasma). Its intensity $I$ will vary according to

$$\frac{dI}{ds} = j(\omega) - I\alpha(\omega)$$

$$I(s) = I(s_0) \exp[-\varepsilon_s] + \frac{j}{\alpha} \left[1 - \exp[E - \varepsilon_s]\right]$$

$$\varepsilon_s = \int_0^s \alpha(\omega) ds$$
is the 'optical thickness'

(All above depend on $\omega$, even if not explicit. Units are $Wm^{-2}sr^{-1}$)

When the $\tau_s>>1$ the medium is optically thick and radiation is in thermal equilibrium with the emissive/absorptive medium and $I(s,\omega) = B(T(s),\omega)$, where

$$B(\omega, T) = \frac{\hbar \nu^3}{2\pi c^2} \cdot \frac{1}{\exp(h\nu/T) - 1} \approx \frac{\nu^2 T}{2\pi c^2}$$
is the blackbody emission (Planck) and the last approximation is for $h\nu<<T$ (Raleigh-Jeans formula), which is appropriate for electron cyclotron emission from fusion plasmas. The black body limit introduces a relation between emissivity and the absorption coefficient (Kirchhoff’s law, true whatever the optical thickness is):

$$\frac{j(\omega, T)}{\alpha(\omega, T)} = B(\omega, T)$$

It follows for the absorption coefficient in a plasma:

$$\alpha_m(\omega, T_e) = \frac{j_m(\omega, T_e)}{B(\omega, T_e)} \propto \left(\frac{T_e}{2m_e c^2}\right)^{m-1}$$
Figure 2: Calculated absorption coefficient for (a) \( \theta = 90^\circ \), relativistic broadening; and (b) \( \theta = 70^\circ \), Doppler broadening (ignoring relativistic effects). Calculations assume typical JET conditions: \( T_e = 10 \text{ keV}, n_e = 2.2 \times 10^{19} \text{ m}^{-3} \) and \( B = 3.4 \text{ T} \).

Figure 3: Calculated absorption coefficient for same conditions as figure 2 plotted against major radius: (a) \( \theta = 90^\circ \), relativistic broadening; and (b) \( \theta = 70^\circ \) Doppler broadening (ignoring relativistic effects).

TCV

(a) Dépendance en température, \( n_e(0) = 2 \times 10^{19} \text{ m}^{-3} \)

(b) Dépendance en densité, \( T_e(0) = 3 \text{ keV} \)

FIG. 3.5 – Représentation de \( \alpha_E^2 \) pour une onde de fréquence \( \omega = 82 \text{ GHz} \). La figure de gauche représente la dépendance du coefficient d’absorption en fonction de la température et à droite en fonction de la densité. La droite verticale en trait-tillé représente la position de la résonance froide.
Optical thickness in a tokamak

When evaluating the optical thickness (or depth) in an inhomogeneous magnetic field, the number of resonant electrons in the path leads to a term proportional to $B/(dB/ds) \propto R$ in a tokamak. For $\theta = \pi/2$:

\[ \tau(\omega) = \int \alpha(\omega) ds \]

In contemporary experiments the first harmonic $O$ mode and the second harmonic $X$-mode are generally optically thick, i.e. $\tau > 3$.

For $X$-mode, as used in TCV, $\tau^X_2 \approx 3.7 \times 10^{-22} n_e T_e B/R \quad [\text{eV}, \text{m}^{-3}, \text{T}, \text{m}]$

---

$$\begin{array}{c|c|c|c|c|c|c}
\hline
m & \text{Mode ordinaire} & \text{Mode extraordinaire} \\
\hline
1 & \tau^O_1 = \frac{\pi}{2c} \left( 1 - \left( \frac{\omega_{pe}}{\Omega_{ce}} \right)^2 \right)^{1/2} \left( \frac{\omega_{pe}^2}{\Omega_{ce}} \right) \left( \frac{k_BT_e}{m_e c^2} \right) R & \tau^X_1 = \frac{\pi}{2c} \left( \frac{\omega_{pe}^2}{\Omega_{ce}} \right) R \\
2 & \tau^O_2 = \frac{\pi}{c} \left( 1 - \left( \frac{\omega_{pe}}{2\Omega_{ce}} \right)^2 \right)^{3/2} \left( \frac{\omega_{pe}^2}{\Omega_{ce}} \right) \left( \frac{k_BT_e}{m_e c^2} \right)^2 R & \tau^X_2 = \frac{\pi}{c} \left( \frac{\omega_{pe}^2}{\Omega_{ce}} \right) \left( \frac{k_BT_e}{m_e c^2} \right) R \\
m \geq 2 & \tau^O_m = \frac{\pi}{2c} \frac{m^{2(m-1)}(m-1)!}{2^{m-1}(m-1)!} \left( 1 - \left( \frac{\omega_{pe}}{m\Omega_{ce}} \right)^2 \right)^{m-1/2} \left( \frac{\omega_{pe}^2}{\Omega_{ce}} \right) \left( \frac{k_BT_e}{m_e c^2} \right)^m R & \tau^X_m = \frac{\pi}{2c} \frac{m^{2(m-1)}(m-1)!}{2^{m-1}(m-1)!} \left( \frac{\omega_{pe}^2}{m\Omega_{ce}} \right)^{m-1} \left( \frac{k_BT_e}{m_e c^2} \right)^m R \\
\hline
\end{array}$$

*see e.g. Bornatici M. et al, Nuclear Fusion 23 (1983) 1153


*P. Blanchard, EPFL thesis 2606

In contemporary experiments the first harmonic $O$ mode and the second harmonic $X$-mode are generally optically thick, i.e. $\tau > 3$. 

For $X$2, as used in TCV, $\tau^X_2 \approx 3.7 \times 10^{-22} n_e T_e B/R \quad [\text{eV}, \text{m}^{-3}, \text{T}, \text{m}]$
Optical thickness contours for modelled profiles in $T_e(0)$-$n_e(0)$ parameter space for $X2$, $\tau_{X2} = \int \alpha_{X2}(R) dR$ for a horizontal view through the TCV plasma midplane:

Fig. 3.8 - Figures représentant les lignes d’équi-valeur de $\tau^X_2(0)$ sur deux grilles $[T_e(0), n_e(0)]$. (a) : Les températures et densités sont plutôt dans la gamme de valeur d’un plasma thermique avec des profils $T_e(r) = T_e(0)(1 - (r/a)^2)^\alpha$ et $n_e(r) = n_e(0)(1 - (r/a)^2)^\beta$, $\alpha = 2$ et $\beta = 0.5$. (b) : Les températures et densités sont plutôt représentatives de la population suprathermique avec des piquages $\alpha = 0.5$ et $\beta = 6$.

Observation of suprathermal/thermal electron populations:

Radiation from each of the population is partly absorbed when it passes through the region occupied by the other population.

Normally the thermal population is thick enough to screen suprathermal radiation for LFS $T_e$ measurements for $\theta=0$. 

VI) Electron Cyclotron Emission 15 April 2009 6
Accessibility: limited by $\varepsilon \to 0$ (cutoff, see reflectometry, chapter 4):

For O1: $\omega > \omega_{pe}$ and for X2 $\omega > \omega_{U} = \frac{\omega_{ce} + \sqrt{\omega_{ce}^2 + 4\omega_{pe}^2}}{2}$

define a critical density above which radiation does not leave the plasma. Depends also on viewing geometry (HFS/LFS).

**Fig. 3.2 - (a)**: Représentation de l’indice réfraction $N^2$ en fonction de la pulsation $\omega$. La courbe en traitillé est relative au mode ordinaire tandis que les courbes en trait plein correspondent au mode extraordinaire. Les fréquences indiquées ont été calculées avec les paramètres typiques de TCV présentés au paragraphe 3.2.4. **(b)** : Diagramme CMA pour les électrons représentant les coupures (•••) et les résonances (---) pour les modes ordinaires et extraordinaires d’une propagation perpendiculaire.

**Fig. 3.3 - (a)** : Distribution spatiale des trois premières harmoniques de $\Omega_{ce}$, des fréquences de coupure $\omega_{pe}$ et $\omega_{R}$ et de la fréquence de résonance $\omega_{nh}$ dans le tokamak TCV. **(b)** : Représentation de la densité critique locale $n_{crut}$ pour la seconde harmonique de $\Omega_{ce}$ due à la coupure $\omega_{R}$ sur TCV avec $B_T(0) = 1.45T$. Le centre du plasma est à $R = 0.89m$.

*P. Blanchard, thesis, 2002*
Localisation

- 95% of the radiation comes from where $\int \alpha ds$ integrates up to 3.
- In the case of high optical thickness ($\tau >> 3$) this can be a narrow layer near the cold resonance for LFS observation.
- For the same frequency, with $\tau >> 3$, HFS and LFS observation correspond to (slightly) displaced observation regions.

Detection of ECE radiation

- Fourier transform spectrometer: large band, can cover 2-3 harmonics
  Michelson interferometer, mechanical scanning of the length of one the arms. Each $\omega$ component produces fringes with different period in $\Delta x$. A Fourier transform of $I(\Delta x)$ provides $E(\omega)$ up to a scale factor.
  Low time resolution (~tens of ms). JET uses an absolutely calibrated instrument of this kind.
- Grating spectrometer: fairly high resolution, good sensitivity
- (Super)-heterodyne: highest sensitivity and resolution. Uses one or two levels of heterodyne mixing to select a frequency band. (used on most fusion devices, including JET and TCV)
• Beam transport:
  - Oversized open waveguide with or without focussing element
  - Fundamental waveguide with low divergence antenna and optional focussing element (mirror) as in TCV (fig. below)
  - Large distances in oversized waveguides to reduce losses

• TCV currently has two fixed-channel radiometers. One is generally used for HFS measurements (sensitive to suprathermal electrons) and the other for LFS. For each, there are two antennae, which are chosen depending on the plasma position. The LFS midplane view is better collimated, because it uses a plastic lens, allowing a narrow focus in the plasma.

• In addition, an orientable mirror system (like for ECH) on the midplane, is available. It could be used for studies of the fast electron population (θ≠0).

• TCV has a correlation ECE system, currently equipped with two frequency scannable YIG filters. They allow to position observation channels at arbitrary distance within the plasma, for correlation studies (MHD, turbulence). See V. Udinstev’s crpplocal pages.
Schematic of one of the super-heterodyne HFS radiometers on TCV. 24 channels, 78.5-114.5 GHz (0.63m<R<0.91m for $B_T=1.43T$) channel bandwidth $\Delta f_{IF}=0.75\text{GHz}$, ‘video’ bandwidth $\Delta f_{V}=40\text{kHz}$.
Sensitivity

- Single mode waveguide has ‘diffraction-limited’ étendue:
  \[ \int A(\theta, \phi)d\Omega = \lambda^2 \text{, whatever the shape of the antenna pattern.} \]
  Can be understood from uncertainty principle \( \Delta x \Delta k \geq 2\pi \) by noting that \( \Omega \geq (\Delta k/k)^2 \).

- If single mode receiver sees only blackbody radiation (B) at temperature \( T >> h\nu \) (Rayleigh Jeans limit), it will receive a power per unit frequency bandwidth \( p(\omega, T) = \int B(\omega, T)A(\theta, \phi)d\Omega = k_B T \)

- A practical radiometer only receives a fraction of this because of coupling losses throughout the system.

- As long as statistical noise from the source dominates over receiver noise, measurement noise is limited by Bose-Einstein statistics of blackbody radiation (exercise 6.1):
  \[
  \frac{\Delta T_{\text{rms}}}{T} = \sqrt{\frac{\Delta f_V}{\Delta f_{\text{IF}}}} \sim 10^{-2} \text{ for typical conditions.}
  \]
Example: Sawteeth in thermal TCV plasma (P. Blanchard, thèse 2602, FSB, 2002)

HFS system, ECH, no ECCD, calibrated against Thomson scattering

\[ \text{Figure 6.3 - (a): Evolution temporelle de signaux ECE de la décharge n° 18876. (1) Canal sur l’axe } r = 0. (2) Canal juste à l’intérieur du rayon d’inversion. (3) Canal sur le rayon d’inversion. (4) Canal en dehors du rayon d’inversion. (b): Représentation sur une vue polaire du TCV de la position spatiale de ces canaux. (c): Agrandissement d’une dent de sée pour deux canaux du radionmètre. Le premier [—] provenant du canal (1) et le second [ - - ] du canal (4). Aux trois temps indiqués nous avons représenté les profils radiaux de température ECE sur la figure (d).} \]
Example: Suprathermal ECE in TCV produced by X2 ECCD and X3 ECE
VII) Particle diagnostics

Heavy Ion Beam Probe

- One of the few diagnostics capable of measuring plasma potential
- Measurements of density and potential fluctuations
- Future: measurements of vector potential → poloidal flux
- Principle: inject singly ionised heavy ions (E~200keV in TJ-II and T10)analyse escaping double-ionised secondaries
Accelerator 100-500keV, <1mA, for small research device, Cs, Tl ...

\[ \text{A}^+ + e^- \rightarrow \text{A}^{2+} + 2e^- \]

Directional energy analyser to observe \( A^q \) from small sample volume provides plasma potential at ionisation location:

\[ W_{\text{sec}} = W_{\text{pri}} + (q_{\text{sec}} - q_{\text{pri}}) \Phi_{sv} \]

Secondary beam intensity related to \( n_e \) at sample volume

\[ I_{\text{sec}} = I_{\text{pri}} n_e \Delta l \sigma_{\text{eff}} q_{\text{sec}} / q_{\text{pri}} \]

\( I_{\text{pri}} \) from attenuation calculation. Relative fluctuations do not require \( I_{\text{pri}} \), but some ‘contamination’ due to line integral effects along trajectories exists.

Nearby (cm) sample volumes provide turbulence \( k, \omega \) spectra by correlation, as well as cross-phase between \( v_r = E_p / B_T \) and \( n_e \) fluctuations.

Toroidal displacement of secondary related to poloidal flux at sample volume.
Primary and secondary beams need to be steered by controlled electrostatic sweep plates to compensate for ambient magnetic fields. Primary beam impact monitored by in-vessel detector

Split energy analyser with 4 quadrants
* control detector input sweep plates from left/right imbalance.
* control high voltage at top plate to keep beam up/down centered.

The high voltage $V_A$ at the top plate allowing beam centering is proportional to the ion energy $W_{sec}$

Analyser can be equipped with several quadrant detectors for correlation measurements

By varying injection energy and direction a large portion of a cross section can be accessed.

![Image of primary beam detector system](image1)

![Image of energy analyzer](image2)
Two-point correlation measurements at \(r_1, r_2\):

* Divide the signals (\(n_e\) or \(\Phi\) fluctuations) into \(j=1\) to \(M\) time windows with preferably \(2^N\) samples
* Fast Fourier Transform on each window \(\rightarrow A_{1j}(\omega)\) and \(A_{2j}(\omega)\)
* Autocorrelation theorem says \(P_\omega(r_1, r_2) = \langle A_{1j}(\omega) \ast A_{2j}(\omega) \rangle_j\) is cross-power spectrum (FT of temporal cross-correlation function). Average taken over the \(M\) time windows.

Figure 9. A detection grid obtained by varying sweep plate and analyzer voltages for Ti in ISX-B, with a toroidal field of 1.2T [3].

Figure 10b. An enlarged view of three adjacent sample volumes [12].

HIBP in TEXT

Wootton & Schoch in Diagnostics for Contemporary Fusion Experiments, Course & Workshop, Varenna 1991
$k$ estimated from 2 point correlations:

$$
\bar{P}_\omega(r_1, r_2) = \bar{S}_\omega \bar{Y}_\omega(r_1, r_2) \exp(i\bar{\phi}_\omega(r_1, r_2))
$$

$S_\omega$ is the geometric mean of the two auto-powers:

$$
\bar{S}_\omega(r) = \left[ \left| \bar{P}_\omega(r_1) \right|^2 \left| \bar{P}_\omega(r_2) \right|^2 \right]^{1/2}
$$

$\gamma_\omega < 1$ is the cross-power coherence:

$$
\bar{\gamma}_\omega(r_1, r_2) = \frac{\left| \bar{P}_\omega(r_1, r_2) \right|}{\left[ \left| \bar{P}_\omega(r_1) \right|^2 \left| \bar{P}_\omega(r_2) \right|^2 \right]^{1/2}}
$$

and $\phi_\omega$ is the cross-power phase:

$$
\bar{\phi}_\omega(r_1, r_2) = \arg \bar{P}_\omega(r_1, r_2)
$$

This allows the wave number $k_\omega$ to be calculated:

$$
\bar{k}_\omega(r_1, r_2) = \frac{\bar{\phi}_\omega(r_1, r_2)}{|r_2 - r_1|}
$$

Example from TEXT:

![Graphs showing phase and coherence](image)

*Figure 15. Typical 2 point density information from TEXT, a) phase and b) coherence. The data is from sample volumes poloidally separated by 2.4cm (solid line) and 4.7cm (broken line). The sample volumes are at r/a = 0.75 [15].*
Same measurements allow local particle flux to be estimated:

$$\Gamma_r = \langle \tilde{n} \tilde{v}_r \rangle = \frac{\langle \tilde{n}(t) \tilde{E}_\phi(t) \rangle}{B}$$

This can be written in terms of the measurable parameters as [13]

$$\Gamma_r = \frac{2}{B} \int_0^\infty \text{Re}[P_{nE}(\omega)] \omega d\omega = \frac{1}{B} \int_0^\infty \tilde{n}_{ms}(\omega) \tilde{\phi}_{ms}(\omega) k_\theta(\omega) |\gamma_{n\phi}(\omega)| \sin(\alpha_{n\phi}(\omega)) d\omega$$

Early TEXT result suggest electrostatic turbulence important for particle transport:

![Figure 16. A typical phase angle between $\tilde{n}$ and $\tilde{\phi}$.

Figure 17. The electrostatic turbulence driven particle flux $\Gamma^{f,E}$ from HIBP and probes, and the total particle flux $\Gamma^i(H_\alpha)$ from spectroscopy.

The right figure shows that the particle flux is important near the edge and drops off sharply towards the interior of the plasma. This is expected since the source of neutrals is located near the edge. Neutrals from outside the plasma cannot penetrate very far.
HIPB measurement of local poloidal flux (dream)

Plasma poloidal field causes a toroidal displacement of the primary and secondary beams (a few cm). Since analyser position is fixed, this has to be compensated for by toroidal steering of the primary beam.

In an axisymmetrical field (tokamak) there is a simple relation between the vector potentials at the injection, ionisation and detection points and the beam momentum. It results from the conservation of canonical momentum:

\[ P_\phi = m R v_\phi + q R A_\phi = \text{constant of motion} \]

\[ B = \nabla \times A, \text{ in particular in a tokamak } B_p = \nabla \times (A_\phi \hat{\nabla}_\phi) \text{ and for the poloidal flux passing between flux surfaces 1 & 2 (full turn definition of } \psi:) \]

\[ \psi_2 - \psi_1 = \int_S \nabla \times (A_\phi \hat{\nabla}_\phi) dS = \oint A_\phi \hat{\nabla}_\phi \cdot d\mathbf{l} = 2\pi (R_2 A_\phi - R_1 A_\phi) \]

Hence conservation of canonical momentum can be written in more familiar terms: \( P_\phi = m R v_\phi + 2\pi q \psi \)

The conservation law relates the source (s), ionisation (i) and detection (d) points by:

\[ m R_s v_\phi + 2\pi q \psi_s = m R_i v_\phi + 2\pi q \psi_i \]

and neglecting the momentum change due to electron impact ionisation, with the charge, 2e , of the secondary ion explicited:

\[ m R_i v_\phi + 2 \cdot 2\pi e q \psi_i = m R_d v_\phi + 2 \cdot 2\pi e \psi_d \]
The two equations combine:

\[ 2\pi \psi_i = \frac{2\pi m}{e} (R_d \nu_{\phi d} - R_s \nu_{\phi s}) + 4\pi \psi_d - 2\pi \psi_s \quad \text{or} \]

\[ \psi_i - \psi_d = \psi_d - \psi_s + \frac{2\pi m}{e} (R_d \nu_{\phi d} - R_s \nu_{\phi s}) \]

- Since there are deflectors between the injection and detection points, additional known terms, depending on the deflection voltages appear in the above equation.

- \( \psi_d \) and \( \psi_s \) do not depend on the precise distribution of current in the plasma and can be provided by a flux map of the entire experiment, using the known coil currents and magnetic measurements.

- \( \nu_{\phi d} \) can be determined from a two-point measurement along the secondary trajectory, for instance at the entrance at at the exit of an energy analyser, allowing simultaneous electrical and magnetic potential measurements.

- A key issue is the effect of the toroidal field ripple (loss of axisymmetry) experienced by the particles as they pass between the toroidal field coils on their ways in and out of the vacuum chamber. However the full 3D magnetic field distribution outside the plasma is known from magnetic diagnostics and hence on trajectories in outside the plasma (on the way in and out) can be calculated in principle exactly. A solution for minimizing ripple may be to make the ions pass very close to the plane of symmetry between adjacent TF coils, where \( dB/d\phi \sim 0 \).

- In the late nineties a reconstruction of the equilibrium using the position at which secondaries arrived after passing the plasma was achieved in TEXT (G J Schwefelberger et al, Rev. Sci. Instrum, 69, 3828 , 1998). This sheme leads to a mixture of local and trajectory-integrated contributions, which must be unfolded.
Momentum balance of fast Tl ions in T10 (conceptual study, Febr. 2008)

\( \beta_1, \beta_2 \) are electrostatic steering plates, which ensure that the ion arrive at the detector with the right angle. They ’donate’ angular momentum to the ions. Ripple is strongest near TF coils, just outside plasma. Below, trajectories with and without ripple.
Figure below shows how the important quantities (poloidal flux $\psi$, angular momentum $l$, canonical momentum $C$ (all expressed in units of flux)), vary along the ion trajectory.

Note the contributions to $l$ provided by the steering plates, note also that when the ripple is included a 'parasitic' contribution to $l$ comes from the ripple region (near TF coils). This leads to an erroneous estimate for $\psi$, denoted $\psi_t$. The meaning of $\psi_t$ is simply an effective $\psi$, calculated from $B_z$ and $B_R$, as encountered by the ion and hence dependent on the ion trajectory, since these two fields contain components produced by the TF coils. $C$ here, in the ripple cases, also has a slightly different meaning, since it includes $\psi_t$, rather than $\psi$.

This effect must be accurately calculated/calibrated for a measurement of axisymetrical part of the poloidal flux, i.e. the part not including contributions from the TF coils. It’s possible, if on the detection side, the ion position is known at two points, for instance. Successful demonstration of local poloidal flux measurements using HIBP would represent a breakthrough.