Abstract. Sawtooth inversion radii and profile peaking factors of a large variety of ohmic and ECH heated L-mode plasmas, including elongations up to 2.6 and triangularities between -0.5 and 0.75 have been investigated in the TCV tokamak. In ohmic plasmas, normalised inversion radii and electron temperature profile peaking factors (corrected for sawtoothing effects) depend solely on the parameter $<j>/q_0 j_0$, irrespectively of plasma shape. With ECH this parameter remains the main scaling parameter. Density profiles are well described as functions of the poloidal flux, in agreement with turbulent equipartition theories. The paper provides parameter conversions allowing the observed scalings to be expressed using the conventional scaling variables $q_{95}$, $\delta_{95}$ and $\kappa_{95}$. 

Shape Dependence of Sawtooth Inversion Radii and Profile Peaking Factors in TCV L-mode Plasmas


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1. Introduction

The dependence of the profiles of plasma parameters, their peaking factors and sawtooth inversion radii on plasma cross sectional shape are of importance for an optimal design of Next Step devices. The optimal design of a Next Step device is a complex task, which requires accurate knowledge of many aspects of plasma behaviour as a function of design parameters [1][2][3]. Inaccuracies or lack of such knowledge may lead to sub-optimal design choices. This is also true of the issue of optimal plasma shaping. Plasma shaping has profound effects both on MHD stability and on confinement. Sawtooth behaviour in particular, may have important repercussions on plasma performance, since sawtooth crashes often provide the seed islands which trigger Neoclassical Tearing Modes (NTM), causing severe performance degradation [4][5][6]. High triangularity improves MHD stability, both of internal kink modes associated with the sawtooth instability [11] and of edge localised modes [4][8][9][10]. The higher current carrying capacity of highly elongated plasmas leads to increased beta limits [12] and confinement, as reflected in the tokamak confinement scaling laws [13]. High elongation also leads to confinement improvements as a result of geometrical effects [14][15].

Experience with strongly shaped tokamak plasmas is limited to a small number of devices [16][17][18]. As a result Next Step designs tend to remain conservative in terms of shaping capability. Frequently expressed concerns with highly elongated designs ($\kappa \geq 2$) include the reduced vertical stability and the fear that the high plasma currents in these devices may lead to excessively large sawtooth inversion radii. It is often implicitly assumed that the corresponding sawtooth crashes would be larger and more likely to trigger NTM’s, than sawteeth at lower elongation.

The TCV tokamak [17][18] ($R_0=0.88m$, $a<0.25m$, $B_T<1.54T$, $I_p<1.2MA$), with its vacuum vessel elongation of 3, is a unique device for experimental investigations of the effects of plasma shaping. Profiles and sawtooth behaviour from a large variety of stationary ohmic and ECH plasmas in TCV have been analysed for presentation in this article. These results, together with those previously published on the shape dependence of sawtooth periods and crash amplitudes [11], show that, contrary to widespread belief, fears of excessively large inversion radii and sawtooth amplitudes at high plasma elongation, are ill founded.
2. Sawtooth inversion radii

The scaling of sawtooth inversion radii in circular discharges is expressed as $r_{inv}/a = 1/q_a$ and has been known for a long time [19]. In strongly shaped discharges $q_a$ however ceases to be a meaningful scaling parameter. Although the safety factor at 95% of the poloidal flux, $q_{95}$, offers an improvement, the best scaling for TCV is given as $\rho_{inv} \propto \langle j \rangle / (q_0 j_0)$, where $\langle j \rangle$ is the cross sectionally averaged toroidal current density, $q_0$ the axial safety factor and $j_0$ the axial current density. The normalised inversion radius is defined as $\rho_{inv} = (A_{inv} / A_p)^{1/2}$ where $A_p$ is the plasma cross-sectional area and $A_{inv}$ is the area of the sawtooth inversion contour. This scaling is shown in Fig. 1 for ohmic plasmas where the effect of sawtooth crashes was measured by a 200 channel soft x-ray tomography system. The identification of the inversion contour involved an SVD analysis of the reconstructed emissivity distribution in order to reject all spatial components (topos) which are not poloidally symmetrical. The dataset shown includes a wide variety of plasmas, the confinement properties of which were already presented previously [15]. Plasma parameters span a wide range: $1 < \kappa_a < 2.6$, $0.5 < \delta_a < 0.7$, $2 < q_{95} < 7$, $1.2 \times 10^{19} \text{ m}^{-3} < \bar{n}_e < 12 \times 10^{19} \text{ m}^{-3}$, $0.1 < \nu^\gamma_{75} < 10$, where $\kappa_a$ and $\delta_a$ are the elongation and triangularity at the last closed flux surface (LCFS) and $\nu^\gamma_{75}$ is the electron collisionality at 75% of the poloidal flux. Elongations are identified by corresponding symbols in the figure, demonstrating adherence to the proposed scaling up to extreme shaping conditions. Since $\langle j \rangle / (q_0 j_0) \propto 1/q_a$ in circular discharges (up to diamagnetic and paramagnetic corrections to the equilibrium field), this parameter provides a generalisation of the $1/q_a$ scaling observed in circular plasmas [19]. This scaling parameter can be evaluated without knowledge of the central current density or safety factor since $q_0 j_0 = B_0(\kappa_0 + L \kappa_0)(\mu_0 R_0)$, where $\kappa_0$ is the elongation at the magnetic axis.

![Fig. 1 Normalised inversion radius from x-ray tomography versus scaling parameter $\langle j \rangle / (q_0 j_0)$ in ohmic discharges. Different symbols refer to different classes of elongation at the LCFS.](image-url)
Inversion radii obtained for a smaller set of L-mode data with central ECH at a level of up to 1.4 MW (up to 4 times the power of the ohmic target discharges) are shown in fig. 2. These experiments were conducted using three gyrotrons operating at a frequency of 82.7 GHz, corresponding to the second EC harmonic in TCV. The confinement of these discharges has been presented previously [20].

The steerable mirrors of the ECH launchers systems in TCV allow ECH to be delivered essentially to any location in the vessel, at any polarization (normally X-mode) and with a current drive component (negligible in these experiments) determined by the toroidal launch angle. Plasma parameters in the ECH dataset also span a wide range: $1.15 < \kappa_a < 2.15$, $-0.4 < \delta_a < 0.7$, $2.1 < q_{95} < 4.6$, $1 \times 10^{19} \text{m}^{-3} < n_e < 3 \times 10^{19} \text{m}^{-3}$, $0.03 < \nu^*_{75} < 0.2$.

Again there is no discernible dependence of inversion radii on elongation, when expressed as a function of $<j>/q_{\phi 0}$, although part of the data fall somewhat short of the ohmic scaling. This may be due either to small changes in the current profiles or to increased peaking of the pre-crash emissivity profiles [21].

A modest, albeit significant, dependence of inversion radii on ECH is observed when deposition just inside or just outside the inversion radius. The former leads to slightly increased inversion radius, while deposition just outside reduces the inversion radius [21]. These effects are attributed to changes in the current profiles resulting from changes in the electron temperature and hence conductivity and current profiles, as well as from a small current drive component due to the magnetic field line pitch. Partial sawtooth stabilization due to fast particles [22][23][24] may also lead to increased inversion radii, but is not important in these X2 ECH experiments, since suprathermal ECE, or hard X-rays from suprathermal electrons, are only detected when a significant current drive component is present [25][26].

![Fig. 2 Normalised inversion radius from x-ray tomography versus scaling parameter $<j>/q_{\phi 0}$ in ECH discharges. Different symbols refer to different classes of elongation at the LCFS.](image-url)
It is important to stress that the sawtooth inversion radius is a poor indicator of the sawtooth crash amplitude when a wide range of plasma shapes or localised ECH is under consideration [11][21][27][28]. Sawtooth amplitudes are much smaller at elongations in excess of 2 and sawteeth sometimes disappear altogether at elongations above 2.2, to be replaced by higher frequency relaxations, which prevent core profiles of temperature, density and X-ray emission from becoming peaked. An example of sawtooth amplitude reduction with elongation is shown in Fig. 3 for an ohmic discharge in TCV, which reached an elongation of 2.8 and had a sawtooth inversion radius close to 60% of the minor radius before the disappearance of sawteeth rendered the determination of the inversion radius impossible using the above-mentioned SVD method. In contrast, high triangularity leads to lengthened sawtooth periods and larger crashes [11][15].

**Fig. 3 Reduction of sawtooth amplitude in an ohmic limiter discharge (no 19373) where elongation was increased up to 2.8.**

top: Central electron temperature from an X-ray filter measurement as a function of time.

middle: Inversion radius from X-ray tomography and SVD sawtooth analysis and scaling parameter $<\jmath>/(<q_0\jmath_0)$ (continuous line).

bottom: Elongation at LCFS.

3. Profile peaking factors

Observations that profiles of plasma temperature, pressure and current tend to adopt universal shapes have been reported and discussed since the early days of tokamak research [19][29][31][32][33][34]. In circular plasmas profile peaking factors are reported to depend on $I/q_a$ [19]. In ohmic TCV plasmas we observe that the profile inverse peaking factors (normalised widths) $<p_e>/p_{e0}$, $<T_e>/T_{e0}$ and $<n_e>/n_{e0}$ for electron pressure, temperature and density, also depend on the current profile peaking via the parameter $<\jmath>/(<q_0\jmath_0)$, irrespective of plasma shape and electron density [35][36]. These observations suggested that the profile
peaking factor scaling results from the 'clipping effect' of sawtooth crashes, which regularly flatten the core profiles up to the inversion radius. The evidence presented here shows however that the profiles in the confinement zone (outside the inversion radius) are also determined by $j/(q_0 j_0)$.

During the sawtooth cycle, core plasma temperatures rise and flatten periodically, while temperatures at and beyond the inversion radius experience only small variations. The result is that inverse peaking factors as defined above depend on the time during the sawtooth cycle at which profile measurements are taken, using, as in TCV, a repetitively pulsed Thomson scattering system [37]. This effect introduces a significant amount of scatter in the data [35][36]. Moreover, for comparison with theoretical models, only the confinement region (outside $\rho_{inv}$) may be considered as ohmically relaxed and close to steady-state. We have therefore introduced “clipped” profile widths such as $\bar{p}_e/p_e$ where $p_{ei}$ is the electron pressure at the sawtooth inversion radius and $\bar{p}_e=p_{e1}$ for $\rho<\rho_{inv}$ and $\bar{p}_e=p_e$ for $\rho \geq \rho_{inv}$. In the region $\rho \geq \rho_{inv}$ profiles change only little during the sawtooth cycle, except for a brief heat pulse following the sawtooth crash. As a result, these “clipped” profile peaking factors are a measure of the shape in the confinement zone (degree of convexity) and can be considered as the idealised profile widths shortly after the sawtooth crashes, when the core profiles are flattened and the heat pulse has abated.

These widths are shown in Fig. 4 and Fig. 6 for electron pressure and temperature in ohmic plasmas and exhibit a remarkably narrow distribution as a function of $j/(q_0 j_0)$. The standard deviations from the fitted lines in the figures are 0.0172 and 0.0173. This parameter performs better than safety factors based scalings such as $l q_{95}$ for which the standard deviations are 0.034 and 0.038, suggesting that the current profile or the electron temperature profile, determine all other profiles. The electron temperature profiles are in good agreement with an ohmic relaxation model based on the assumption that the magnetic entropy of the discharge is constant in time, which predicts a rigid current profile [38] depending essentially only on $j/(q_0 j_0)$.

Although no detailed comparisons of this theory with ECH plasmas are available to date, the same rigid current profiles as in ohmic plasmas are predicted, provided no ECH is deposited in the confinement zone. A very similar scaling is indeed observed in ECH heated L-mode plasmas, as shown in Fig. 5 and Fig. 7, although the data are more dispersed, due to a lower signal-to-noise ratio of the Thomson scattering measurements at low density. The
relative insensitivity of profile shapes to heating power may also be interpreted in terms of critical temperature gradient models [39].

Fig. 4 Scaling of electron temperature inverse peaking factors (confinement zone only) in ohmically heated TCV discharges. Symbols refer to different classes of plasma elongation at the LCFS. Fit: $\bar{T}_e/T_{e1} = 0.92\langle j \rangle/(q_0 j_0) + 0.13$.

Fig. 5 Scaling of electron temperature inverse peaking factors for ECH dataset. Symbols refer to different classes of plasma elongation at the LCFS. Fit: $\bar{T}_e/T_{e1} = 0.77\langle j \rangle/(q_0 j_0) + 0.17$.

Fig. 6 Scaling of electron pressure inverse peaking factors for ohmic dataset. Symbols refer to different classes of plasma triangularity at the LCFS. Fit: $\bar{p}_e/p_{el} = 1.04\langle j \rangle/(q_0 j_0)$.

Fig. 7 Scaling of electron pressure inverse peaking factors for ECH dataset. Symbols refer to different classes of heating power. Fit: $\bar{p}_e/p_{el} = \langle j \rangle/(q_0 j_0)$.
Electron density profiles are on average slightly broader in ECH discharges (Fig. 9) than in OH discharges (Fig. 8) and the corresponding peaking factors are also more dispersed than those for the temperature and pressure profiles. Remarkably, the density profiles in both the OH and the ECH dataset exhibit a simple, universal dependence on poloidal flux, seen in Fig. 10 and Fig. 11, which is in agreement with Turbulent Equipartition Theories (TEP), [41][42][43][44]. The essential prediction of these theories is that, as a result of the (conjectured) approximate conservation of the magnetic momentum $\mu_B$ and of the longitudinal adiabatic invariant $J$, during turbulent transport, particles with given values of these two invariants tend to spread evenly over the accessible poloidal flux, i.e. $\partial N_{\mu_B,J}/\partial \Psi \cong \text{const.}$ This leads to density profiles expressed as

$$n_{\mu_B,J} = \frac{\partial N_{\mu_B,J}}{\partial V} = \frac{\partial \Psi}{\partial V} \cdot d\Phi/dV \propto 1/q \cdot d\Phi/dV$$

The factor $d\Phi/dV$ is equal to $B_0/(2\pi R_0)$ at the plasma centre and increases in TCV to typically 1.25 times that value at the plasma edge, as a result of finite aspect ratio and shaping, mainly triangularity. If particles with all pitch angles (trapped and passing) are transported equivalently, the above equation is applicable to the overall density profile. If the relative transport of passing particles is less than that of trapped particles, the predicted overall density profile can be approximated by $n_{\mu_B,J} = \frac{\partial N_{\mu_B,J}}{\partial V} = \frac{\partial \Psi}{\partial V} \cdot d\Phi/dV \propto 1/q \cdot d\Phi/dV$ with $0.3 \leq \eta \leq 1$ [45]. In Fig. 10 and Fig. 11 we plot the integrated, normalized electron density in the confinement zone, for the same database samples as the previous figures, as a function of poloidal flux normalized to its value at the LCFS. The linear relationship in Fig. 10 shows that the particle content within any flux interval is indeed proportional to the poloidal flux within that interval, as expected for $\eta \cong 1$. The ECH L-mode profiles in Fig. 11 correspond on average to $\eta \cong 0.8$. This value is in agreement with observations in L-mode plasmas in the DIII-D tokamak [45]. The prediction for ‘clipped’ widths as a function of $<j>/(q_0\Phi_0)$, for $\eta=1$ and $0.8$ are within a few percent of the empirical fits shown in Fig. 8 and Fig. 9 respectively [46]. Within the framework of TEP, the broader profiles with ECH correspond to increased turbulent diffusion of trapped particles.

One should note that density profiles depart from TEP predictions in the case of ECH heating in the confinement zone and in all cases in the vicinity of the LCFS. Possible reasons are the non conservation of the above mentioned invariants in the presence of intense sources of heat and the influence of electrostatic potentials, which enter in the definition of $J$. 

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but are not considered in the derivation of TEP predictions. Particle expulsion from the plasma core (‘pumpout’), as observed with strong central ECH in the presence of a (1,1) island, is also at odds with TEP expectations, presumably because the low levels of turbulent particle diffusivity in the central region allow neoclassical effects to compete [47].

An alternative explanation of the observed density profile behaviour is offered by turbulent thermodiffusion. This process is based on an energy dependent phase space diffusivity of the particles and is expected to lead to steady-state density gradients which, in source-free regions and in the absence of other pinch mechanisms, are proportional to the temperature gradients, such that \( \nabla n/n = -\alpha T \nabla T/T \), with \(-0.5<\alpha_T<0\) [48][49]. Applied to the electrons for \(\alpha_T=-0.5\), the prediction is in good agreement with observations for \(\langle j \rangle/(q_0j_0)\approx0.4\), but predicts significantly broader profiles than observed for \(\langle j \rangle/(q_0j_0)<0.3\). A potentially relevant comparison with ion temperature profiles is not available, although ion temperature profiles are believed to be less peaked than electron temperature profiles, due to the absence of direct ion heating in TCV. Density profiles a hence unlikely to be determined by thermodiffusion alone.

\[
\frac{n}{n_e} \propto -0.5 <\alpha_T<0.5
\]

Fig. 8 Electron density profile inverse profile factors in OH discharges. Symbols refer to categories of triangularity.

Fit: \(n_e/n=n_e^{2.46}\langle j \rangle/(q_0j_0)^2+2.57\langle j \rangle/(q_0j_0)-0.014\)

Fig. 9 Electron density profile inverse peaking factors in ECH discharges.

Fit: \(n_e/n_e=0.74\langle j \rangle/(q_0j_0)+0.39\)
4. Scaling expressions for ohmic target plasmas

Although auxiliary heating and energetic particles can influence both the inversion radius and profile peaking factors, it is useful to be able to predict these parameters for an ohmic target discharge of arbitrary shape. The scaling parameter \( \langle j \rangle (q_0 j_0) \) can be evaluated using an equilibrium code. Alternatively it can be approximated to a high accuracy using global discharge parameters as shown below. First we factorise as follows:

\[
\langle j \rangle = \langle j \rangle^* \cdot \frac{2}{\kappa_0 + 1 / \kappa_0} \cdot \frac{B_{0\text{vac}}}{B_0},
\]

(eq.1)

where \( \langle j \rangle^* \) is the dimensionless average current density, \( \langle j \rangle^* = \frac{\mu_0 R_0 \langle j \rangle}{B_{0\text{vac}}} \equiv \frac{2}{q_{\text{eng}}^2} \), (eq.2)

and \( \langle j \rangle = I_p / A_p \), is the cross-sectional average current density. The toroidal field correction, which can be fitted as

\[
B_0 / B_{0\text{vac}} \approx 1 + 0.054 \cdot (1 - \beta_p) \langle j \rangle^*^2
\]

(eq.3), is unimportant for all but the highest current densities, which are only obtained at extreme elongations [18]. The core elongation correction in eq.1 can be related to the corresponding
edge factor in terms of $\kappa_{95}$ by the empirical fit:

$$\frac{(\kappa_0 + 1/\kappa_0)}{(\kappa_0 + 1/\kappa_{95})} \approx 1 - 0.09\kappa_{95}(\kappa_{95} - 1)
\left(1 - \frac{\langle j \rangle^2}{4} - \frac{\beta_p}{4}\right).$$  \hspace{1cm} (eq. 4)

For scaling purposes using the conventional database variables $q_{95}$, $\delta_{95}$ and $\kappa_{95}$, the dimensionless average current density has been fitted over the TCV database.

For limiter plasmas with $\delta_{95}>0$ we obtain

$$\langle j \rangle^* \approx 0.92 \frac{f(\varepsilon)}{f_{TCV}} \left(\kappa_{95} + 1/\kappa_{95}\right)(1 + 2\delta_{95}^2)(1 + 0.14\kappa_{95})/q_{95} \right\},$$  \hspace{1cm} (eq.5)

and for Single Null diverted plasmas with parameters ($0.2<\delta_{95}<0.45$ and $1.5<\kappa_{95}<2.2$) encompassing those of the current ITER-FEAT design:

$$\langle j \rangle^* \approx 0.98 \frac{f(\varepsilon)}{f_{TCV}} \left(\kappa_{95} + 1/\kappa_{95}\right)(1 + 0.7\delta_{95}^2)(1 + 0.14\kappa_{95})/q_{95} \right\},$$  \hspace{1cm} (eq.6),

where $f(\varepsilon) \equiv (1.17 - 0.65\varepsilon)/(1 - \varepsilon^2)$ is an aspect ratio correction [50] and $\varepsilon_{TCV}=0.27$.

5. Elongation scaling of plasma current for fixed inversion radius

One of the benefits of high elongation in a large future fusion experiment is the ability to raise the plasma current for a given value of edge safety factor. From eq.1, using the observed scaling $\rho_{inv} \equiv \langle j \rangle / q_0\langle j \rangle_0$, we are able to express the total plasma current as a function of normalised inversion radius and elongation:

$$I_p \approx \rho_{inv} \cdot \frac{\pi a^2 B_0}{\mu_0 R_0} \cdot \kappa_{95}(\kappa_0 + 1/\kappa_0)$$  \hspace{1cm} (eq.7)

where $a$ is the minor radius and $\kappa_a$ is the elongation at the last closed flux surface. This relation is similar to the one obtained when scaling $I_p$ at fixed safety edge factor $q_{95}$, where $I_p \propto q_{95}^{-1}\kappa_{95}(\kappa_{95} + 1/\kappa_{95})$. If sawtooth inversion radii are a concern, the design plasma current and elongation can be scaled for a fixed inversion radius, with a small penalty (compared to fixed safety factor scaling), given by the ratio expressed in eq. 4. For typical plasma parameters this ratio is still as high as 0.9 for $\kappa_{95}=2$. 
6. Conclusions

In ohmic shaped plasmas the parameter \(<j>/q_{i0}\) determines the inversion radius, as well as temperature and density profiles in the confinement zone, irrespectively of plasma cross sectional shape. This is also the dominant scaling parameter in centrally heated ECH plasmas. Electron temperature profiles may be interpreted by global constraint based models [38][46] or critical gradient models [39]. Density profiles in the experiments reported here appear to follow expectations from turbulent equipartition theories [41]-[46].

These observations, which were carried out over an extremely wide range of plasma shapes, including elongations up to 2.54, show that fears of excessively large inversion radii and sawtooth crashes in highly elongated discharges are ill founded. Discharges may be designed for a given normalized inversion radius, rather than any particular value of edge safety factor. Most of the current carrying capacity resulting from high elongation is retained, even when optimizing a design with the constraint of a fixed inversion radius. The experimental observations of reduced sawtooth periods and amplitudes at high elongation also favour high elongation [11]. One should however beware of premature conclusions concerning the impact at high elongation of sawteeth on “knock-on” MHD such as NTMs or ELMs, since the stability of these modes is also likely to depend on plasma shape. Such issues, as well as the effect of fast particles on sawteeth at high elongation, still await experimental investigation.

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