Turbulent Particle transport in Magnetized Plasmas
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Abstract: Particle transport in magnetized plasmas is investigated with a fluid model of drift wave turbulence. An analytical calculation shows that magnetic field curvature and thermodiffusion drive an anomalous pinch. The curvature driven pinch velocity is consistent with the prediction of turbulence equipartition theory. The thermodiffusion flux is found to be directed inward for a small ratio of electron to ion pressure gradient, and reverses its sign when increasing this ratio. Numerical simulations confirm that a turbulent particle pinch exists. It is mainly driven by curvature for equal ion and electron heat sources. The sign and relative weights of the curvature and thermodiffusion pinches are consistent with the analytical calculation.

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Particle transport is a key issue in magnetically confined fusion plasmas since the fusion power increases with density. The ionization source will be mainly peripheral in a future reactor. Therefore the density gradient is expected to be small on the basis of diffusive transport. However the density profile is often peaked in tokamaks, even when the fuelling is peripheral. This observation is traditionally translated into a particle flux of the form $\Gamma_s = -D_s \nabla n_s + V_s n_s$, where $V_s$ is the pinch velocity, $D_s$ the diffusion coefficient and $n_s$ the density of the species ‘s’. The theory of collisional transport shows that the inductive field in a tokamak induces a pinch of electrons, the Ware pinch [1]. Also transport analysis of tokamak plasmas indicates that the diffusion coefficient $D_s$ is usually anomalous, i.e. larger than the collisional value. This anomaly is attributed to turbulent diffusion. A pending question is whether the pinch velocity is collisional or anomalous. Experimental results are quite contradictory regarding this issue. Density profiles were found to be consistent with a Ware pinch only in Asdex-U [2] and JET [3] for plasmas at high density in the H-mode. On the other hand, an anomalous pinch has been observed in various devices in the L-mode [4,5,6], including JET [7]. From the theory standpoint, two mechanisms leading to an anomalous pinch have been proposed. One is based on turbulent...
A set of 5 fluid equations is used here to describe a collisionless ITG/TEM turbulence

$$d_t n_e = i\omega_d n_e [\hat{p}_e \phi - \hat{T}_e n_e] + S_n,$$  
$$d_t p_e = i\omega_d [\hat{p}_e \phi + \hat{T}_e n_e - 2\hat{T}_e p_e] + S_{pe},$$  
$$d_t \Omega = -\n_e \nabla x v_{//} - i\omega_d n_e [\hat{p}_e \phi + p_i]$$  
$$-i\omega_d f_i [n_e \phi - p_e] + \left[\hat{p}_i, \nabla^2 \phi\right] + f_c \left[\hat{n}_e, \nabla^2 \phi\right]$$  
$$d_t v_{//} = -\nabla x (\phi + p_i/\hat{n}_e) + S_v,$$  
$$d_t p_i = -i\omega_d [\hat{p}_i (1 - f_c \hat{T}_i / \hat{T}_e) \phi - \hat{T}_i^2 n_e + 2\hat{T}_i p_i]$$  
$$-\Gamma \hat{p}_i \nabla x v_{//} + S_{pi}$$

where $n_e$, $T_e$, $p_e$, $v_{//}$, $\phi$ are the normalized density, temperature, pressure, parallel velocity and electric potential (the labels ‘e’ and ‘i’ are for electrons and ions, no impurity is included). The generalized vorticity $\Omega$ is defined as $\Omega = \hat{n}_e \left[f_c (\phi - \langle \phi \rangle) / \hat{T}_e - \nabla^2 \phi\right]$. The normalization is of the gyroBohm type, $n_e \rightarrow a / \rho_{so} n_e / n_0$, $p_{ei} \rightarrow a / \rho_{so} p_{ei} / p_i$, $\phi \rightarrow a / \rho_{so} e \phi / T_{ei}$, $v_{//} \rightarrow a / \rho_{so} v_{//} / c_{so}$, where $\rho_{so}$ is the ion gyroradius ($\rho_{so} = m_{so} e / e B_{0}$), $c_{so}$ is the sound speed $(T_{ei} / m_{so})^{1/2}$, $a$ and $R$ are the minor and major radius, $n_0$, $T_0$, $p_0 = n_0 T_0$ are reference values. Time and spatial co-ordinates are normalized to $a / c_{so}$ and $\rho_{so}$. The geometry of flux surfaces is circular concentric, $(r, \theta, \varphi)$ being the labels of the minor radius, poloidal and toroidal angles ($p = a / a$ is the normalized minor radius). The fraction of trapped (resp. passing) electrons is $f_e = 2 / \pi (2r/R)^{1/2}$ (resp. $f_i = 1 - f_e$). The electron precession drift and the ion curvature drift operators are respectively $\omega_d = \mp 2 e_\lambda \rho_{so} r^{-1} \hat{\partial} \varphi$ and $\omega_d = -i 2 e_\rho \rho_{so} (\cos(\theta) r^{-1} \hat{\partial} \theta + \sin(\theta) \hat{\partial} \varphi)$. The function $\lambda = 1 / 4 + 2 s / 3$ characterizes the dependence of the precession frequency on the magnetic shear $s = pdq / dq \rho$ and $e_\lambda = a / R$ parameters the curvature. The Lagrangian time derivative is defined as $d_t = \hat{\partial} + [\phi, ] - D$, where $D$ is a “collisional” diffusion operator and $[f, g] = r^{-1} (\hat{\partial} f \hat{\partial} g - \hat{\partial} f \hat{\partial} g)$. The functions $S_{n}$, $S_{v}$, $S_{pe}$, $S_{pi}$ are particle, momentum, ion and electron heat sources. A hat indicates a flux average. Note that the perturbed part of $f_i n_e$ is the fluctuating density of trapped electrons, whereas $\hat{n}_e$ is the total equilibrium electron density. The adiabatic compression index is $\Gamma = 5 / 3$.

The vorticity equation (1c) expresses an ambipolarity condition. The vorticity is coupled via the curvature drifts to electron and ion pressure, which are governed by Eqs. (1b) and (1e).
parallel momentum equation (1d) is responsible for the slab ITG instability. Since the perturbed electron density \( n_e \) appears in subdominant contributions in the electron and ion heat equations, the subset Eqs.(1b-1e) is quasi-autonomous for small equilibrium density gradient. When pressure fluctuations are small \((p_e \approx 0)\), Eq.(1a) can be recast as \( d_i(Hn_e)=0 \), where \( H=\exp[\varepsilon_a^p\rho(1/2+4s/3)] \). Thus \( Hn_e \) behaves as a passive scalar in this case. If the transport due to velocity fluctuations is diffusive, the "natural" density profile is proportional to \( 1/H \), in agreement with the TEP prediction [10-12]. Also electron pressure fluctuations are small when the electron pressure gradient is weak. Hence trapped electrons behave as "test particles" if the turbulence is mainly driven by ITG modes.

A quasi-linear particle flux can be calculated using Eqs(1a) and (1b),

\[
\Gamma_e = -f_t D_{ql} \left\{ \partial_p \hat{n}_e + 2\varepsilon_a^p \lambda_t \hat{n}_e - 4\varepsilon_a^p \lambda_t V_{phe} \partial_p \hat{p}_e \right\}
\]

where

\[
D_{ql} = \sum_{k_{\theta}0} \frac{k_{\theta}^2}{\Delta\omega_k} |\phi_{k0}|^2 \quad (3)
\]

\[
V_{phe} = \left\{ \omega / k_{\theta}^0 - 2\varepsilon_a^p \Gamma_t \hat{T}_e \right\} \{1\}
\]

\[
\langle F \rangle = \frac{1}{D_{ql}} \sum_{k_{\theta}0} \frac{k_{\theta}^4}{\Delta\omega_k} |\phi_{k0}|^2 F \quad (4)
\]

Here \( \Delta\omega_k \) is a turbulent frequency broadening and \( k=(k_\theta, k_\phi) \) labels the poloidal and toroidal wave numbers. The calculation is done at order one in \( \omega/\Delta\omega_k \sim \mathcal{O}(\varepsilon_a^{1/2}) \). The "phase velocity" \( V_{phe} \) is a shifted poloidal phase velocity \( \omega/k_{\theta}^0 \) averaged over the turbulence spectrum. The expression (2) indicates that both curvature and thermodiffusion pinches appear in this turbulence model. As expected the TEP result is recovered for zero electron pressure gradient since \( dH/Hd\rho=2\varepsilon_a^p \lambda_t \).

We stress here that the particle pinch velocity depends on the magnetic shear via the precession drift frequency of trapped electrons. For an arbitrary geometry, the generalized peaking factor is \( d\hat{n}_e/\hat{n}_e d\psi = -\varepsilon_{de}/\hat{T}_e \), where \( \psi \) is the poloidal flux and \( \omega_{de} \) is the precession frequency averaged over the phase space.

In fact the phase velocity \( \langle \omega/k_{\theta}^0 \rangle \) cannot be chosen freely in Eq.(2). It is constrained by the ambipolarity condition. This property is illustrated here by writing a simplified equation for the ion density using Eqs(1a) and (1c) and the electro-neutrality condition \( n_i=\hat{n}_e f_c(\phi-\langle\phi\rangle)/\hat{T}_c+f_i n_e \). The ion flux is similar to Eq.(2) (with the transformation \( f_t \rightarrow 1, \lambda_t \rightarrow 1, \partial_p \hat{p}_e \rightarrow \partial_p \hat{p}_i \)) in the limit of vanishing parallel wave vector \( k_{//} \) and finite Larmor radius effects, assuming strongly ballooned modes. Accounting for the parallel dynamics leads to an additional contribution \( \langle (k_{//}/k_{\theta}^0)^2 \rangle \partial_p \hat{p}_i \) in the thermodiffusion flux. The average phase velocity \( \langle \omega/k_{\theta}^0 \rangle \) is determined by equating the electron and ion fluxes. It is then used to recalculate the (now equal) ion and electron fluxes,
\[ \Gamma_e = \Gamma_i = -f_i D_{qi} \frac{1 + \lambda_t \tau_e}{1 + f_i / \lambda_i \tau_e} \left\{ \frac{\partial \rho \hat{n}_e + 2 \varepsilon_a \lambda_i}{1 + \lambda_i \tau_e} \hat{n}_e + \left(8 \Gamma \varepsilon_a^2 (1) \left( \lambda_i \hat{T}_e + \hat{T}_i \right) - \frac{\left( k^2 / k_0^2 \right)}{1 + \lambda_i \tau_e} \right) \right\} \]

where \( \tau_e = \frac{\partial \rho \hat{P}_e}{\partial \rho \hat{P}_i} \). The structure of Eq.(5) is similar to Eq.(2). In the limit of strong ion heating \( \tau_e \to 0 \), the pinch velocity due to curvature is identical to Eq.(2), and the TEP result is recovered. Thermodiffusion induces an inward pinch if the average parallel wave number \( k_{/a} \) is large enough. In the opposite limit \( \tau_e >> 1 \), the curvature pinch velocity is controlled by the ion curvature drift \( \nu = 2 \varepsilon_a \). Thus the curvature driven pinch depends at least on the ratio \( \tau_e \) of the electron to ion pressure gradients. A recent analysis indicates that it also depends on the collisionality [15]. The thermodiffusion flux is directed outward if the electron temperature is large enough. This change of sign is due to a change of direction of the average phase velocity. The latter result depends on the closure assumption in the electron and ion pressure equations (parameter \( \Gamma \)), and on the statistical properties of the turbulence (brackets). Also passing electrons are not included here and may also affect the thermodiffusion flux [9].

Eqs(1a-1e) have been simulated with the spectral TRB code [16]. Numerical details are given in the reference [17]. All simulations were done for a normalized gyroradius \( \rho_{s0} / a = 7.510^{-3} \) with an aspect ratio \( R / a = 3 \), typical of a JET plasma. The perturbed fields are set to zero at \( r=a \), whereas the equilibrium edge density and temperature were set to \( \hat{n}_a = 0.3 \) and \( \hat{T}_a = 0.1 \). The profiles of safety factor, heat and particle sources are shown in Fig.1. Electron and ion heating sources are equal, \( S_{pe} = S_{pi} = 0.01 \). The simulations have been run over 12000 time units, i.e. 3 energy confinement times. In these simulations, the fluxes are fixed rather than the gradients. The particle flux is thus maintained to zero so that any density peaking is an unambiguous signature of a turbulent pinch (the Ware pinch is not implemented in this code). Particle flux and density profiles are shown in Fig.1. The density gradient is finite in the region of zero flux, thus giving evidence of a turbulent pinch. To clarify the nature of this pinch, the equation (1a) has been replaced by the equation

\[ d_t n_e = i \Omega_{de} \left( \hat{n}_e X_{Vq} \phi - X_{VT} P_e \right) + S_n, \]

where \( X_{Vq} \) and \( X_{VT} \) are adjustable coefficients. Following the quasi-linear calculation above, setting \( X_{Vq} = 0 \) should suppress the curvature driven pinch whereas \( X_{VT} = 0 \) should suppress thermodiffusion (in the test particle approximation). The results are shown in Fig.2. It is found that \( X_{Vq} = 0 \) enforces a flat density profile, whereas \( X_{Vq} = 2 \) enhances the peaking by a factor close to 2. Conversely setting \( X_{VT} = 0 \) affects weakly the density profile. Electron and ion pressure profiles remain almost the same. This analysis shows that curvature is the main drive for particle pinch when \( S_{pe} = S_{pi} \). The case \( X_{VT} = 0 \) agrees fairly well, as expected, with the TEP profile \( 1 / H \) normalized at \( \rho = 0.8 \) to the profile for \( X_{VT} = 0 \). We note however that the density profile is slightly hollow when \( X_{Vq} = 0 \), which is the indication of a (small) outward thermodiffusion flux. To assess the effect of thermodiffusion, the ratio of ion to electron heating has been changed at constant ion heating source. The density profiles are shown in figure 3 for 3 values of \( S_{pe} / S_{pi} = 0.5, 1 \) and 2. In
this set of simulations, the electron pressure profile increases whereas the ion pressure remains mostly unchanged. In the case of dominant ion heating, the profile is more peaked than expected on the basis of a TEP theory alone. This indicates that an inward thermodiffusion pinch takes place as predicted by Eq.(5) for large enough values of \( \langle k_{\parallel}/k_{\theta} \rangle^2 \). The density profile becomes flatter with increasing electron heating. In fact an outward pinch is observed in the edge, consistently with the outward thermodiffusion driven by the electron pressure gradient found in the expression Eq.(5). It is also consistent with the previous test \( X_{\nu_q}=0 \). This outward pinch is only visible in the limit of a large ratio of electron to ion pressure gradient \( \tau_e=3 \). The Probability Distribution Function (PDF) of turbulent flux for \( S_{pe}/S_{pi}=2 \) is shown in Fig.4. It is calculated with a sample of \( 10^4 \) values taken at \( \rho=0.3 \), i.e. far from the particle source. The average is -0.2 and the variance 0.12 (in units of \( 10^{-4} \)). This figure suggests that the pinch comes from many events of all sizes (bulk of the PDF) and not from a few exceptional events that would appear in the tail of the distribution.

In conclusion a clear evidence of an anomalous particle pinch has been found when using a fluid model of ITG/TEM turbulence in a tokamak plasma. The pinch velocity due to field curvature agrees with the TEP prediction in the limit of small electron pressure gradient. Also, turbulence simulations indicate that the density profile is close to the TEP prediction for equal electron and ion heating sources. Thermodiffusion plays an increasing role when changing the ratio of electron to ion temperatures. The corresponding flux is inward for a dominant ion heating and becomes outward when the ratio of electron to ion temperatures is large enough. The latter result depends on the turbulence statistical properties and on the closure assumption. We note that electron density profiles remain peaked in devices where electron heating is dominant [4,5]. This suggests that the outward component may not be sufficient to overcome the curvature pinch, albeit it may explain a reduction of profile peaking.

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**Figure captions:**

- **Fig.1:** Profiles of safety factor, heat source and collisional diffusion coefficient (top panel). Lower panel: time average density profiles, sum of diffusive and turbulent fluxes, particle flux calculated from the source.

- **Fig.2:** Density profiles when suppressing \( (X_{\nu_q}=0) \) or doubling \( (X_{\nu_q}=2) \) the coupling with the electric potential or when suppressing the coupling with the electron pressure equation \( (X_{\nu_T}=0) \). The reference case \( (X_{\nu_q}=1,X_{\nu_T}=1) \) is also shown. The dotted line is the TEP prediction with the same boundary edge profile.

- **Fig.3:** Density profiles when varying the ratio of electron to ion heating \( S_{pe}/S_{pi}=0.5,1 \) and 2. The corresponding values of \( \tau_e \) at \( \rho=0.5 \) are indicated.
Fig. 4: Probability Distribution Function of the turbulent particle flux at $\rho = 0.3$ in linear (top) and log-linear (bottom) scales.

Fig. 1

Fig. 2
Fig. 3

Fig. 4